This document is designed to assist North Carolina educators in effective instruction of the new Common Core State and/or North Carolina Essential Standards (Standard Course of Study) in order to increase student achievement. NCDPI staff are continually updating and improving instructional tools to better serve teachers. **These Graphic Organizers are in draft form and are undergoing review for feedback.**

## *Graphic Organizers SAMPLES*

**What is the purpose of this tool?**

These visual representations assist the student in organizing abstract "big picture" information that is new, overwhelming or misunderstood. Research supports the utilization of graphic organizers as a contributing factor in improving student performance. The examples are to model varied organizers and their use in provoking student engagement, organization, and understanding, thus equipping the teacher with the knowledge to develop and use such tools effectively with the new standards. By providing these examples, DPI is creating a foundation for teachers to shift the focus from merely classroom instruction, but also on student learning.

**What is in the tool?** The instructional tool uses straightforward (clear) models/examples that can be used in the classroom in alignment with the new standards. The tool speaks directly to teachers and holds high expectations for teachers' ability to understand the use of the tool as a mechanism for differentiating instruction.

**How do I send Feedback?**

We intend the explanations and examples in this document to be helpful and specific. That said, we believe that as this document is used, teachers and educators will find ways in which the tool can be improved and made even more useful. Please send feedback to us at feedback@dpi.nc.gov and we will use your input to refine our instructional tool. Thank You!

**Where are the new Common Core State and North Carolina Essential Standards?**

All standards are located at http://www.ncpublicschools.org/acre/standards/

# Use of Number Lines in CCSS

In CCSS the number line is first explicitly addressed in second grade Measurement and Data.  References occur throughout middle school as well as in high school Statistics and Probability. The number line constitutes a unifying and coherent representation for the different sets of numbers which other models cannot do.

The many uses of the number line were introduced only a few years ago, in part resulting from research at the Freudenthal Institute. Most teachers are familiar only with the traditional number line with the whole counting numbers already in place.  Using this model to actually build conceptual understanding of a variety of standards will be challenging.  Therefore, we suggest our graphic organizers be limited to exploring the many uses of the number line, as explicitly called for in the Common Core.

**First Grade**

**1.OA.2** Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

There are cookies on the plate. There are 4 oatmeal raisin cookies, 5 chocolate chip cookies, and 6 gingerbread cookies. How many cookies are there total?

I used a number line.  First I jumped to 4, and then I jumped 5 more.  That's 9. Then I jumped 1 more to make 10.  Then, I jumped 5 more and got 15.  There are 15 cookies on the plate.
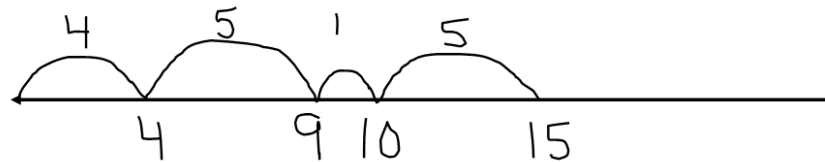
**1.NBT.4** Add within 100, including adding a two-digit number and a one-digit number, and adding a two digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

There are 37 people on the playground. 23 more people show up. How many people are now on the playground?

I used a number line. I started at 37.  Then I broke up 23 into 20 and 3 in my head. Next, I jumped 3 to get to 40. I then jumped 10 to get to 50 and 10 more to get to 60. So, there are 60 people on the playground.
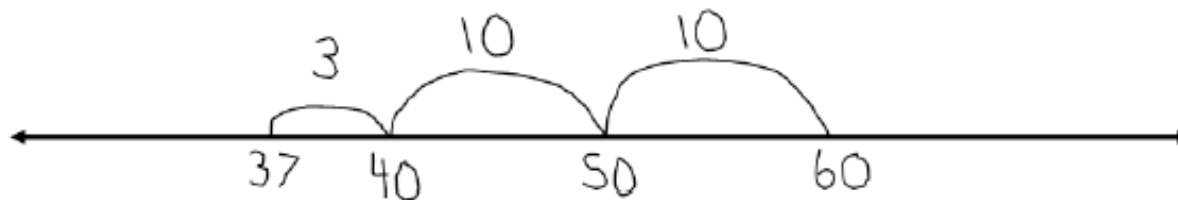
**1.NBT.6** Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

There are 60 students in the gym. 30 students leave. How many students are still in the gym?

I used a number line. I started at 60 and moved back 3 jumps of 10 and landed on 30.
There are 30 students left.
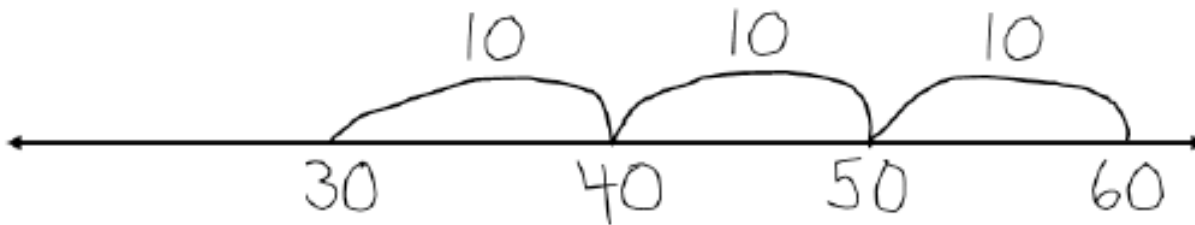
# Using Number lines with 3-5 Common Core State Standards in Mathematics

## Third grade:

**3.NBT.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

178 + 225 = ?
178 + 200 = 378
378 + 20 = 398
398 + 5 = 403
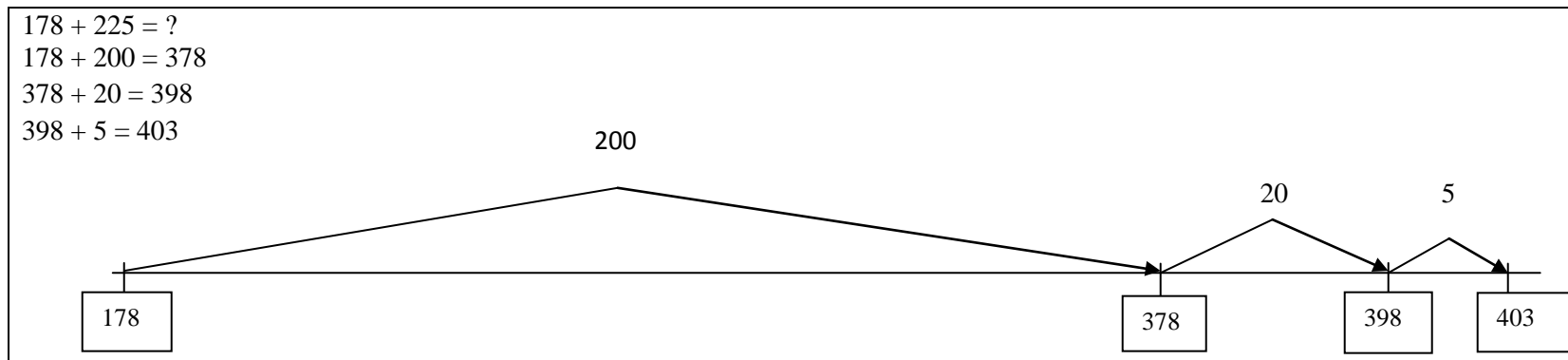
**Third grade:**

**3.NF.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram.
a.  Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
b.  Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

In the number line diagram below, the space between 0 and 1 is divided (partitioned) into 4 equal regions. The distance from 0 to the first segment is 1 of the 4 segments from 0 to 1 or ¼ (**3.NF.2a**). Similarly, the distance from 0 to the third segment is 3 segments that are each one-fourth long. Therefore, the distance of 3 segments from 0 is the fraction ¾ (**3.NF.2b**).
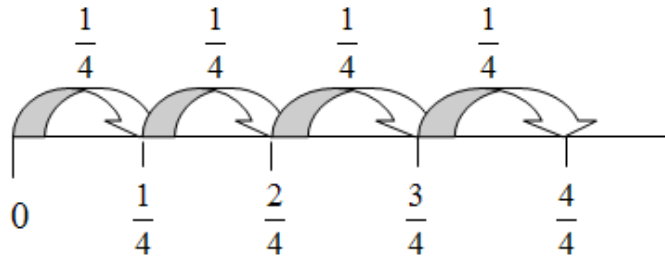


**Third Grade:**

**3.MD.1** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

Example:
Tonya wakes up at 6:45 a.m. It takes her 5 minutes to shower, 15 minutes to get dressed, and 15 minutes to eat breakfast. What time will she be ready for school?
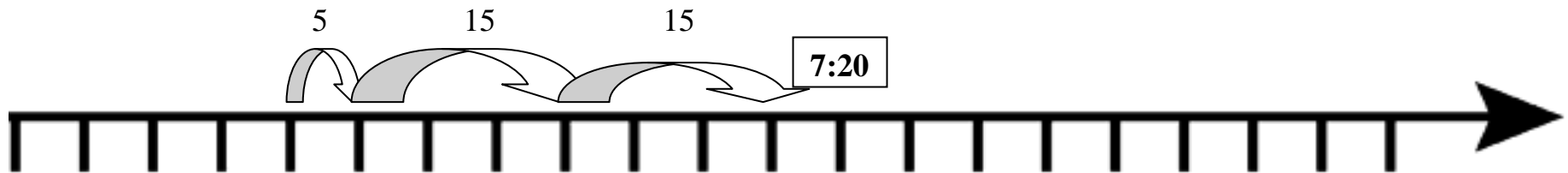
# Fourth Grade:

**4.NBT.3** Use place value understanding to round multi-digit whole numbers to any place.

Round 368 to the nearest hundred.
This will either be 300 or 400, since those are the two hundreds before and after 368.
Draw a number line, subdivide it as much as necessary, and determine whether 368 is closer to 300 or 400.
Since 368 is closer to 400, this number should be rounded to 400



**4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

There are two cakes on the counter that are the same size. The first cake has ½ of it left. The second cake has 5/12 left. Which cake has more left?
Number Line model:

*First Cake*



*Second Cake*
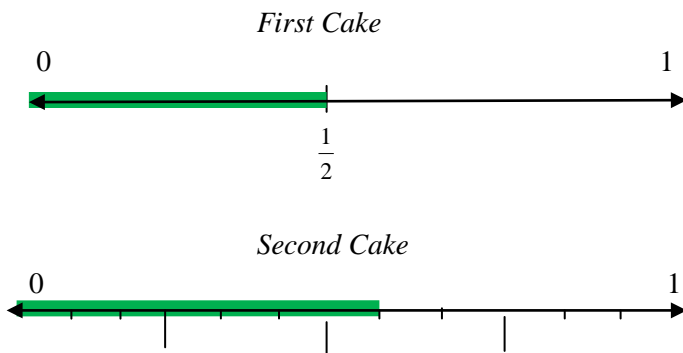
$$\frac{3}{12} \qquad\qquad \frac{6}{12} \qquad\qquad \frac{9}{12}$$

**4.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
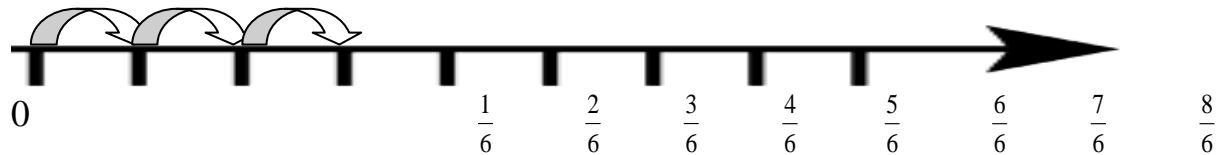
**a.** Understand a fraction *a/b* as a multiple of 1/*b*.

*For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*

This standard builds on students' work of adding fractions and extending that work into multiplication.

3/6 = 1/6 + 1/6 + 1/6 = 3 x (1/6)

Number line:



$$0 \qquad\qquad\qquad\qquad \frac{1}{6} \quad \frac{2}{6} \quad \frac{3}{6} \quad \frac{4}{6} \quad \frac{5}{6} \quad \frac{6}{6} \quad \frac{7}{6} \quad \frac{8}{6}$$

**c.** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

In a relay race, each runner runs ½ of a lap. If there are 4 team members how long is the race?

Draws a number line shows 4 jumps of ½

**½**        **½**        **½**        **½ jump**



**0**      **½**      **1**      **1½**      **2**      **2½**      **3**
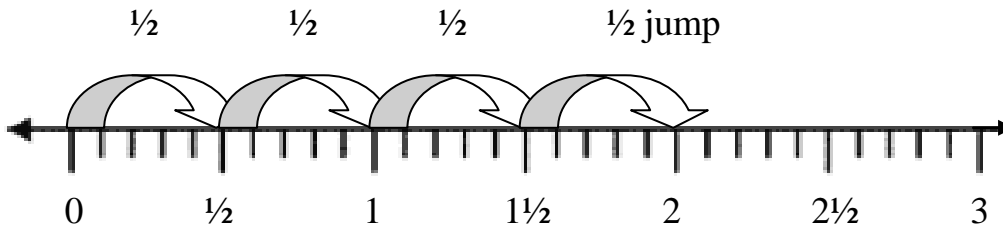
**4.NF.6** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

Students represent values such as 0.32 or 32/100 on a number line. 32/100 is more than 30/100 (or 3/10) and less than 40/100 (or 4/10). It is closer to 30/100 so it would be placed on the number line near that value.

0.32

| 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
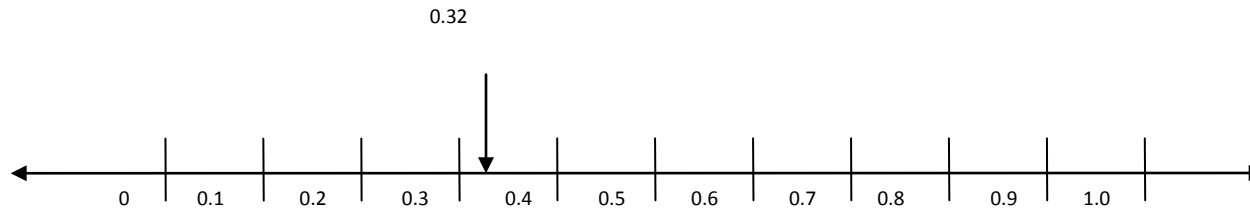
**4.MD.2** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

At 7:00 a.m. Candace wakes up to go to school. It takes her 8 minutes to shower, 9 minutes to get dressed and 17 minutes to eat breakfast. How many minutes does she have until the bus comes at 8:00 a.m.? Use the number line to help solve the problem.

| 6:30 | 6:45 | 7:00 | 7:15 | 7:30 | 7:45 | 8:00 |

# Fifth Grade:

**5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

I saw that the 0.25 in 1.25 and the 0.75 for water would combine to equal 1 whole.
I then added the 2 wholes and the 0.40 to get 2.40.

**.25  +  .75      +      1      +  .40      = 2.40**

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 50 | 1 | 1.5 | 2 | 2.5 | 3 |
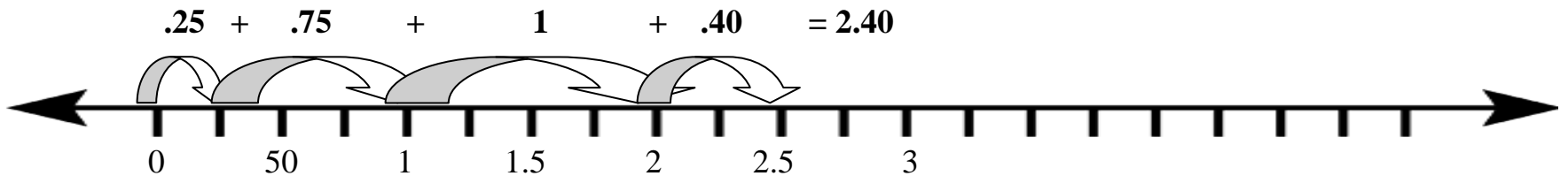
Example of division: finding the number of groups.
Students could draw a segment to represent 1.6 meters. In doing so, s/he would count in tenths to identify the 6 tenths, and be able identify the number of 2 tenths within the 6 tenths. The student can then extend the idea of counting by tenths to divide the one meter into tenths and determine that there are 5 more groups of 2 tenths.

```
|——————1.6 m——————|

|—————————|++++++++|+++|
   1 m        1.6 m  2 m

⊕⊕⊕⊕⊕⊕⊕⊕|+++++|
   1 m        1.6 m  2 m
```
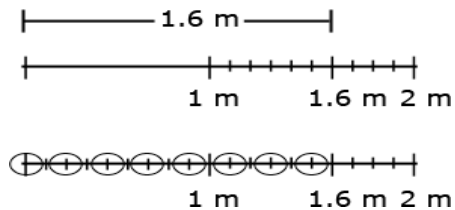
**5.NF.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Jerry was making two different types of cookies. One recipe needed 3/4 cup of sugar and the other needed 2/3 cup of sugar. How much sugar did he need to make both recipes?

- Linear model
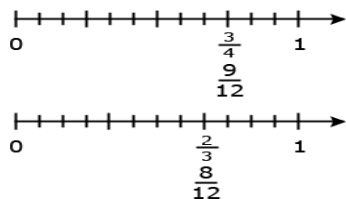


Solution:
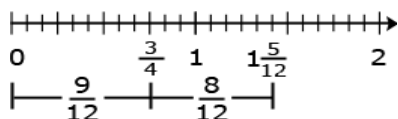


**5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a.  Interpret the product $(a/b) \times q$ as a parts of a partition of $q$ into $b$ equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*

Three-fourths of the class is boys. Two-thirds of the boys are wearing tennis shoes. What fraction of the class are boys with tennis shoes?
This question is asking what 2/3 of ¾ is, or what is 2/3 x ¾.  What is 2/3 x ¾, in this case you have 2/3 groups of size ¾ ( a way to think about it in terms of the language for whole numbers is 4 x 5 you have 4 groups of size 5.
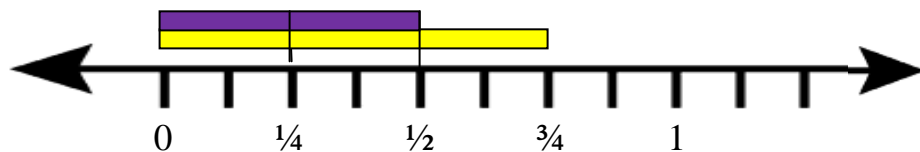
**5.NF.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

a.  Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.
    *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

This standard asks students to work with story contexts where a unit fraction is divided by a non-zero whole number. Students should use various fraction models and reasoning about fractions.

You have 1/8 of a bag of pens and you need to share them among 3 people. How much of the bag does each person get?

Expression 1/ 8 ÷ 3
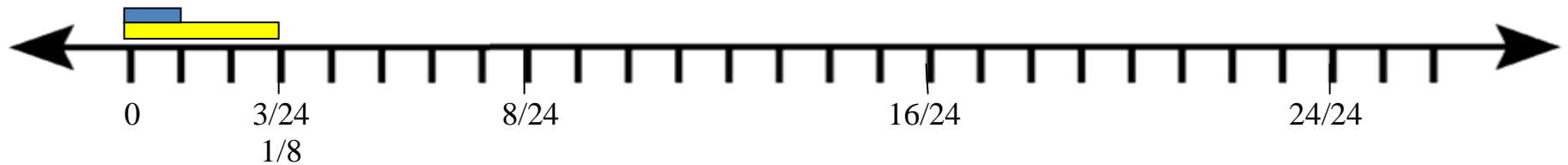
**Using the Number Line to Illustrate Addition of Integers**

7.NS.1
**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**
1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
b.   Understand $p + q$ as the number located a distance $|q|$ from $p$, in the positive or negative direction depending on whether $q$ is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real world contexts.

**Example 1:**
Use a number line to illustrate 6 + (-2)
Step 1:  Move from 0 to the right (positive) a distance of 6
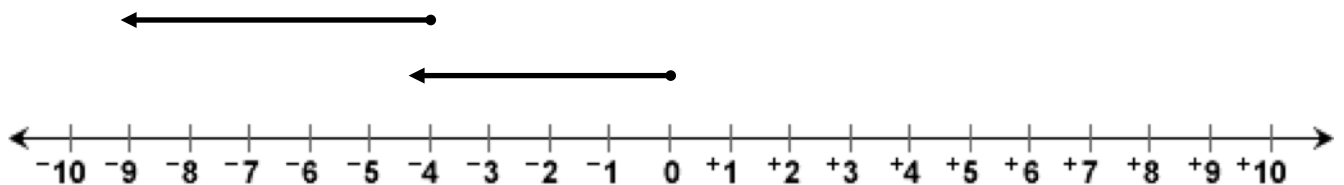Step 2:  From the 6 move a distance of 2 to the left (negative) giving an answer of -4
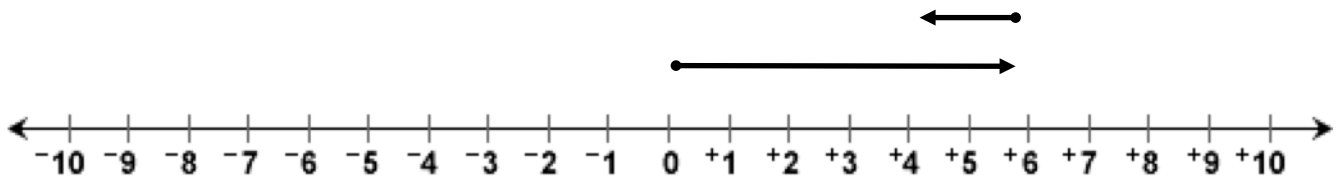
**Example 2:**
Use a number line to illustrate -4 + (-5)
Step 1:  Move from 0 to the left (negative) a distance of 4
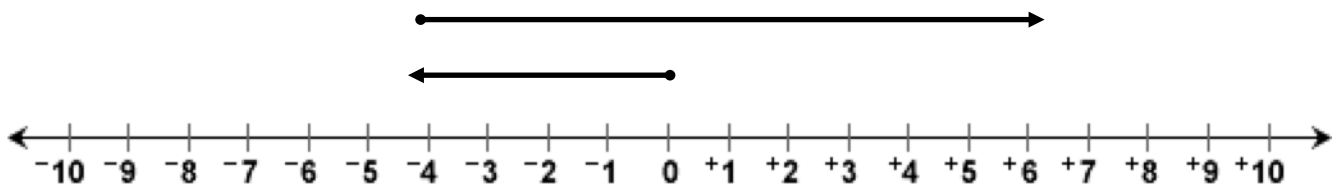Step 2:  From the -4 move a distance of 5 to the left (negative) giving an answer of -9

**Example 3:**
Use a number line to illustrate -4 + 10
Step 1:  Move from 0 to the left (negative) a distance of 4
Step 2:  From the -4 move a distance of 10 to the right (positive) giving an answer of 6

# Solving Absolute Value Inequalities Using The Number Line

**Algebra II 2003 NC Standard Course of Study**
**Use equations and inequalities with absolute value to model and solve problems; justify results.**
**a. Solve using tables, graphs, and algebraic properties.**
**b. Interpret the constants and coefficients in the context of the problem.**

In high school, the number line can and should be used to promote the understanding of solving absolute value inequalities. When setting up the absolute value inequality to solve, the number line is used to determine the direction of the possible values and whether the problem is a conjunction or a disjunction.

For example:

$$|2x - 9| + 20 \le 27$$
$$|2x - 9| \le 7$$

The new inequality indicates that the value inside the absolute value is 7 units or less away from 0. This distance can be 7 units or less on the right side of 0 or 7 units or less on the left side of 0. This situation is represented on the number line below:



Which indicates the setup of the problem and that the solutions for x are:
$$2x - 9 \le 7 \;\; and \;\; 2x - 9 \ge -7$$
$$2x \le 16 \quad and \;\; 2x \ge 2$$
$$x \le 8 \qquad and \;\; x \ge 1$$

We also use the number line to show all the solutions because there are infinitely many, as you will see below.